Chain-to-Chain Competition Under Demand Uncertainty

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Abstract

In this paper, we aim to study the structure choice of supply chains under competitive environment with uncertain demand. We consider two competing supply chains, each of which chooses to either vertically integrate or decentralize with coordinating contracts. We first analyze firms' strategic behavior under given supply chain structures: two integrated chains (II), two decentralized chains (DD), and a mixed structure with one decentralized chain and one integrated chain. We then compare different supply chain structures and examine the equilibrium structure choice. We find that the equilibrium structure depends on the product characteristics. For substitutable products, DD is the equilibrium supply chain structure choice, whereas for complementary products, II is the equilibrium structure. Furthermore, a high demand uncertainty strengthens these equilibrium choices.

Keywords: Chain-to-chain competition; Demand uncertainty; Supply chain coordination; Vertical integration; decentralization.

1. Introduction

In many industries, the competition between firms has become chain-to-chain or channel-to-channel competition, in which an upstream or downstream firm in one supply chain not only competes with its counterpart in the other chain but also interacts with the entire other chain. Chain-to-chain competition is seen in a variety of industries such as fast food, fashion clothing, automobiles, electronics, and telecom industries. For these industries, how a supply chain is structured—vertically integrated or decentralized—critically impacts the supply chain's performance in the competition. This paper aims to analyze how the choice of supply chain structure affects firms' strategic behaviors and what supply chain structure may emerge as an equilibrium choice.

We consider the inventory competition between two competing supply chains. Each chain has one manufacturer and two exclusive retailers. The two manufacturers produce differentiated products, which can be either substitutable or complementary. Each retailer faces random demand and, before the demand is realized, the retail inventory decision must be made by the retailer (if its supply chain is decentralized) or by the entire integrated chain. A retailer's inventory affects the probability distributions of the demands at all retailers. For substitutable products, a retailer's inventory stimulates its own demand while having demand-stealing effect on all other retailers; for complementary products, a retailer's inventory stimulates (instead of stealing) the competing chain's demand. The two chains engage in inventory competition by making newsvendor type of decisions. Our setting may be applied to capacity investment competition, where a high service capacity has demand-stealing effect on other service providers.

When a chain is integrated, the inventory decision is made by the entire chain; when a chain is decentralized, each retailer makes its own inventory decision, but all firms in the chain can coordinate their decisions via supply contracts. We examine three possible supply chain structures: two integrated chains, two decentralized chains, and the mixed structure with one integrated chain and one decentralized chain. These structures are also compared with the first-best structure where one decision-maker decides inventory for both chains. The key questions are: How do firms compete under a given supply chain structure? Which structure outperforms other structures in terms of supply chain profitability, and under what conditions? What is the equilibrium supply chain structure choice? What is the impact of demand uncertainty on the supply chain structure choice?

The chain-to-chain or channel-to-channel competition has been studied in the analytical marketing literature. McGuire and Staelin (1983) analyze various retail distribution structures in the

context of two competing manufacturers, each selling products through an independent retailer. They explain that a strategic reason for manufacturers to use intermediaries is to shield themselves from possibly ruinous price competition. Coughlan (1985) extends this research to a more general demand function and applied it to the electronic industry. Moorthy (1988) links the value of decentralization to the concept of strategic interaction. Zhao, Atkins, and Liu (2009) endogenize the product differentiation and examine the supply chain structure under both price and service competition. There are also models for the industry settings where retailers carry an assortment of products from multiple manufacturers. Choi (1991) considers two manufacturers selling their products through a common retailer. Lee and Staelin (1997) and Trivedi (1998) generalize the above work to a competitive environment with two manufacturers and two common retailers. We also analyze the competition among manufacturers and (exclusive) retailers, but we focus on a setting where demands are uncertain and retail inventories must be decided prior to the realization of the demands.

Supply chain contracts and industry structure choices under demand uncertainty have been considered in the literature. Kranton and Minehart (2000) compare vertical integration versus networks. (Vertically integrated firms make their own inputs, while firms in networks procure inputs from several suppliers who, in turn, sell to several network firms.) They model demand uncertainty as incorporated in firms' random valuation of the supply, and there is no inventory decision in their framework. They show that firms are likely to form networks when they experience large idiosyncratic demand shocks. Ai, Chen, and Ma (2012) examine a setting where manufacturers in two competing supply chains may choose either a wholesale price contract in which the retailer's demand forecasting information is not shared, or a revenue-sharing contract in which the retailer's demand forecasting information is shared. They find that supply chain contract choice depends on the degree of product substitution and demand uncertainty.

Competition among newsvendors has been studied in the literature. Parlar (1988) models a duopoly of two newsvendors selling substitutable products. When either newsvendor experiences stockout, a fraction of the excess demand switches to the other. Lippman and McCardle (1997) start with aggregate industry demand and specify a variety of rules to split the realized aggregate demand among firms. Mahajan and van Ryzin (2001) derive the demand from dynamic consumer choice. There are also simple allocation rules that lead to more explicit solution. Cachon (2003) considers proportional demand allocation rule and reaches qualitatively consistent insights.

In a competitive environment, inventory has demand-stealing effect and demand-stimulating effect. Cachon (2003) finds that "Competition makes the retailers order more inventory because

of the demand-stealing effect: each retailer ignores the fact that ordering more means the other retailers' demands stochastically decrease. ... Furthermore, if retailers sell complements, rather than substitutes, then the demand-stealing effect is reversed: each retailer tends to understock because it ignores the additional demand it creates for other retailers." Dana and Petruzzi (2001) point out that consumers are more likely to purchase from stores that have fewer stockouts. They provide a consumer choice model in which consumers maximize expected utility by taking into account the inventory available at the firm. In this paper, we model the demand distribution as a linear function of the inventory levels. This linear approximation allows us to model both demand-stealing and stimulating effects as well as both substitutable and complementary products. Furthermore, the research efforts in the literature suggest the difficulty in obtaining explicit formulas for equilibrium analysis. This paper aims to explicitly derive and compare equilibria under various supply chain structures, and our demand model makes such analysis tractable.

In this paper, we do not consider price competition nor transshipment between retailers. For models with both price and inventory competition, see Bernstein and Federgruen (2005), and for models with transshipment after the realization of demand, see Anupindi, Bassok and Zemel (2001).

2. The Model

We analyze two competing supply chains, indexed by $i = 1, 2$, with chain i supplying product i. The two products can be either substitutable or complementary. Chain i has one manufacturer, referred to as manufacturer i, and two exclusive retailers, indexed by (i, j), $j = 1, 2$. As a convention, we use $-i$ to refer to the other chain/manufacturer/product, and use $(i, -j)$ to refer to the other retailer of chain i.

We consider a single-period setting where both supply chains produce and stock their products before the selling season starts. Let c_i denote manufacturer i's per-unit production cost, and let v_i denote the per-unit salvage value (if $0 < v_i < c_i$) or disposal cost (if $v_i < 0$) at the end of the selling season. The retail prices for the two products, p_1 and p_2, are assumed to be exogenous.

2.1 Demand Model

The two chains compete by deciding inventory levels before the selling season. Let q_{ij} denote the inventory level at retailer (i, j) and let $\boldsymbol{q} = (q_{11}, q_{12}, q_{21}, q_{22})$. Let random variable ξ_{ij} represent retailer (i, j)'s *natural demand*, which is the demand occurring at the retailer (i, j) when no retailer carries

any inventory ($\boldsymbol{q} = 0$). The random variables ξ_{ij}'s may be dependent. For analytical simplicity, we assume the demands facing retailers in the same chain, ξ_{i1} and ξ_{i2}, have the same probability distribution, with cumulative distribution function denoted as $F_i(x)$, $i = 1, 2$.

The probability distribution of the demand at any retailer is affected by the inventory available at all retailers. To facilitate the analysis, we assume these effects have a linear form. Specifically, the random demand facing retailer (i, j) is modeled as

$$D_{ij}(\boldsymbol{q}) \;=\; \xi_{ij} + \alpha_i q_{ij} - \beta_i q_{i,-j} - \frac{1}{2}\gamma_{-i}(q_{-i,1} + q_{-i,2}), \qquad i, j = 1, 2, \tag{1}$$

where

$$1 > \alpha_i \geq \beta_i \geq 0, \quad \beta_i \geq \gamma_i, \quad 1 > \alpha_i - \beta_i - \gamma_i \geq 0, \quad \text{for } i = 1, 2, \quad \text{and} \quad \gamma_1\gamma_2 \geq 0. \tag{2}$$

In model (1), inventory levels shift the natural demand in the following ways:

- The term $\alpha_i q_{ij}$ suggests that inventory availability stimulates own retailer's demand.
- The term $-\beta_i q_{i,-j}$ reflects a within-chain demand-stealing effect: the availability of product i at a retailer reduces the demand at the other retailer of the same chain.
- The last term $-\frac{1}{2}\gamma_{-i}(q_{-i,1} + q_{-i,2})$ captures the between-chain demand-stealing effect when $\gamma_1 > 0$ and $\gamma_2 > 0$ (i.e., substitutable products) or demand-stimulating effect when $\gamma_1 < 0$ and $\gamma_2 < 0$ (i.e., complementary products).

The assumption of $\alpha_i \geq \beta_i$ in (2) indicates that raising inventory level q_{ij} does not reduce own chain's demand. We refer to $\alpha_i - \beta_i$ as inventory's *net* stimulating effect on own chain's demand.

Because γ_1 and γ_2 have the same signs (assumption of $\gamma_1\gamma_2 \geq 0$), we use $\gamma > 0$ and $\gamma < 0$ to indicate substitutable and complementary products, respectively, throughout the paper. For substitutable products, $\beta_i \geq \gamma_i > 0$ in (2) implies that the demand-stealing effect is stronger within chains than between chains, because the retailers of the same chain carry the same product.

The *total* effect of inventory q_{ij} on all demands is $\alpha_i - \beta_i - \gamma_i$, assumed to be in $[0, 1)$ in (2). That is, one unit of inventory does not reduce the total demand, nor does it stimulate more than one unit of demand.

We assume $q_{ij} \in [0, q_{ij}^{\max}]$ and denote the strategy set as $Q = \{\boldsymbol{q} : q_{ij} \in [0, q_{ij}^{\max}]\}$. Throughout the paper, we assume $D_{ij}(\boldsymbol{q}) \geq 0$, for $\boldsymbol{q} \in Q$ and all equilibria are attained within the interior of Q.

The assumption of linear effects of inventory on demand allows us to explicitly analyze the competition between the two chains and derive useful insights.

2.2 Service Level and Inventory

Let S_{ij} denote the service level of retailer (i, j). That is,

$$S_{ij} = \mathsf{P}\big[D_{ij}(\boldsymbol{q}) \le q_{ij}\big] = F_i\Big((1 - \alpha_i)q_{ij} + \beta_i q_{i,-j} + \frac{1}{2}\gamma_{-i}(q_{-i,1} + q_{-i,2})\Big), \qquad i, j = 1, 2. \tag{3}$$

Assume $F_i(x)$ is continuous and strictly increasing in x for $F_i(x) \in (0, 1)$. Thus, $F_i^{-1}(y) = \inf\big\{x :$ $F(x) \ge y\big\}$ is continuous and strictly increasing in y for $y \in (0, 1)$. Then, (3) leads to a system of linear equations for $\boldsymbol{q}$:

$$(1 - \alpha_i)q_{ij} + \beta_i q_{i,-j} + \frac{1}{2}\gamma_{-i}(q_{-i,1} + q_{-i,2}) = F_i^{-1}(S_{ij}), \qquad i, j = 1, 2. \tag{4}$$

For given retail service levels $S_{ij} \in (0, 1)$, $i, j = 1, 2$, the system of equations in (4) uniquely determines the inventory level $\boldsymbol{q}$. In particular, if the two retailers in chain i achieve service level S_i, i.e., $S_{i1} = S_{i2} = S_i$, for $i = 1, 2$, then system in (4) determines the corresponding inventory levels:

$$q_{i1} = q_{i2} = \frac{1}{\eta}\Big[(1 - \alpha_{-i} + \beta_{-i})F_i^{-1}(S_i) - \gamma_{-i}F_{-i}^{-1}(S_{-i})\Big], \qquad i = 1, 2, \tag{5}$$

where $\eta = (1 - \alpha_1 + \beta_1)(1 - \alpha_2 + \beta_2) - \gamma_1\gamma_2$. Note that the relations in (2) imply that $\eta > 0$.

3. Supply Chain Structures and Strategic Interactions

This section analyzes the firms' strategic behaviors under various supply chain structures. Four structures are analyzed and illustrated in Figure 1.
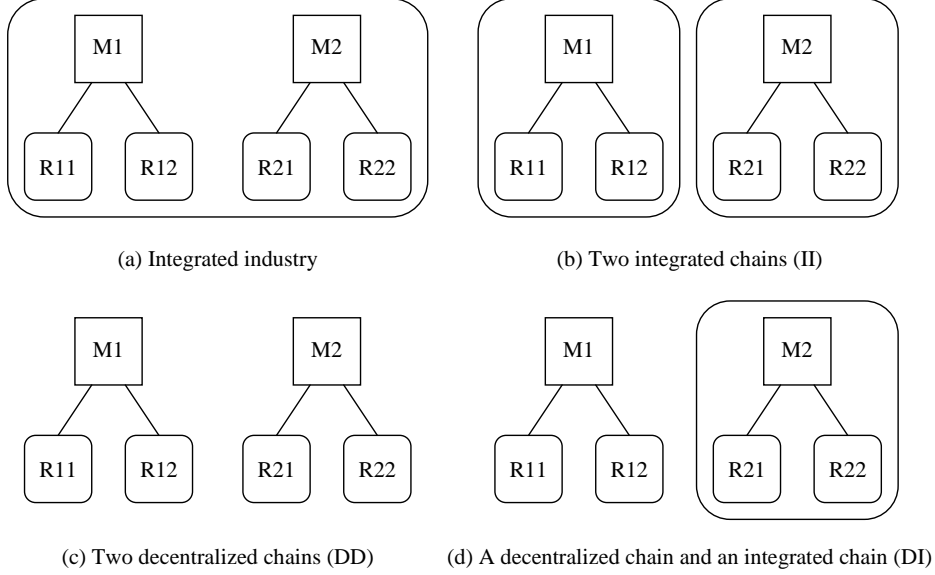
(a) **Integrated industry**: The entire industry is integrated and decision-making is centralized.

(b) **II**: Each manufacturer is integrated with its own retailers.

(c) **DD**: Both chains are decentralized.

(d) **DI**: One chain is integrated and the other is decentralized.

3.1 Integrated Industry

We first consider a benchmark case where an integrated industry centrally decides the stocking levels at all retailers to maximize the total expected profit. Throughout the paper, we use $\sum_{i,j}$ as an abbreviation for $\sum_{i=1}^{2}\sum_{j=1}^{2}$, use $\sum_i$ for $\sum_{i=1}^{2}$, and use $\sum_j$ for $\sum_{j=1}^{2}$. The industry aims to maximize the

Figure 1: Industry Structures

M = Manufacture,   R = Retailer

(a) Integrated industry

(b) Two integrated chains (II)

(c) Two decentralized chains (DD)

(d) A decentralized chain and an integrated chain (DI)

expected total profit:

$$\max_{\boldsymbol{q}} \sum_{i,j} \Big[(p_i - c_i)q_{ij} - (p_i - v_i)\mathsf{E}\big[(q_{ij} - D_{ij}(\boldsymbol{q}))^+\big] \Big]. \tag{6}$$

Note that $(q_{ij} - D_{ij}(\boldsymbol{q}))^+$ is jointly convex in $\boldsymbol{q}$ because $D_{ij}(\boldsymbol{q})$ is linear in $\boldsymbol{q}$. Hence, the objective in (6) is jointly concave in $\boldsymbol{q}$.

Theorem 1 *The industry optimal service levels and inventory levels are given by $S_{ij} = S_i^*$ and $q_{ij} = q_i^*$, for $i, j = 1, 2$, where*

$$S_i^* \stackrel{\text{def}}{=} \frac{p_i - c_i - \frac{\gamma_i(p_{-i} - c_{-i})}{1 - \alpha_{-i} + \beta_{-i}}}{(p_i - v_i)(1 - \alpha_i + \beta_i - \frac{\gamma_1 \gamma_2}{1 - \alpha_{-i} + \beta_{-i}})}, \tag{7}$$

$$q_i^* \stackrel{\text{def}}{=} \frac{1}{\eta}\Big[(1 - \alpha_{-i} + \beta_{-i})F_i^{-1}(S_i^*) - \gamma_{-i}F_{-i}^{-1}(S_{-i}^*) \Big]. \tag{8}$$

The resulting optimal profit of the industry is

$$\pi^* = \sum_i \Big[2(p_i - c_i)q_i^* - 2(p_i - v_i)\int_0^{F_i^{-1}(S_i^*)} F_i(u)du \Big]. \tag{9}$$

Furthermore, S_i^* and q_i^* increase in $\alpha_i - \beta_i$ and decrease (increase) in $\alpha_{-i} - \beta_{-i}$ if $\gamma > 0$ ($\gamma < 0$). The total profit π^* increases in $\alpha_i - \beta_i$ and decreases in γ_i.

Note that if $\alpha_i = \beta_i = \gamma_i = 0$, then $S_i^* = (p_i - c_i)/(p_i - v_i)$, the standard newsvendor's service level. The monotonicity properties of S_i^* and q_i^* in Theorem 1 reveal that if inventory's net stimulating

effect on own chain's demand, $\alpha_i - \beta_i$, increases for product i, the industry should raise product i's inventory and service level, and reduce (raise) the substitutable (complementary) product's inventory and service level.

The monotonicity properties of π^* also justify our intuition that strong net demand-stimulating effect (high $\alpha_i - \beta_i$) and weak product substitution or strong product complementarity (low γ) are favorable conditions for industry profitability.

3.2   Two Integrated Chains (II)

In this structure, each manufacturer vertically integrates with its exclusive retailers. The two integrated chains engage in inventory competition. Given chain $(-i)$'s inventory levels $q_{-i,1}$ and $q_{-i,2}$, chain i's best response problem is

$$\max_{q_{i1}, q_{i2}} \sum_j \Big[(p_i - c_i) q_{ij} - (p_i - v_i) \mathsf{E} \big[(q_{ij} - D_{ij}(\boldsymbol{q}))^+ \big] \Big]. \tag{10}$$

The first-order condition for (10) leads to chain i's optimal service level, which in turn determines the best response in terms of inventory levels. The equilibrium inventory levels can then be found, which is also stated in Theorem 2.

Theorem 2 (II equilibrium) *When each chain is vertically integrated, the equilibrium service levels and inventory levels are given by $S_{ij} = S_i^{\mathrm{II}}$ and $q_{ij} = q_i^{\mathrm{II}}$, for $i, j = 1, 2$, where*

$$S_i^{\mathrm{II}} \stackrel{\text{def}}{=} \frac{p_i - c_i}{(p_i - v_i)(1 - \alpha_i + \beta_i)}, \tag{11}$$

$$q_i^{\mathrm{II}} \stackrel{\text{def}}{=} \frac{1}{\eta} \Big[(1 - \alpha_{-i} + \beta_{-i}) F_i^{-1}(S_i^{\mathrm{II}}) - \gamma_{-i} F_{-i}^{-1}(S_{-i}^{\mathrm{II}}) \Big]. \tag{12}$$

Chain i's profit in the eqilibrium is

$$\pi_i^{\mathrm{II}} = 2(p_i - c_i) q_i^{\mathrm{II}} - 2(p_i - v_i) \int_0^{F_i^{-1}(S_i^{\mathrm{II}})} F_i(u) du. \tag{13}$$

Furthermore, S_i^{II}, q_i^{II}, and π_i^{II} increase in $\alpha_i - \beta_i$; q_i^{II} and π_i^{II} decrease (increase) in $\alpha_{-i} - \beta_{-i}$ if $\gamma > 0$ ($\gamma < 0$).

Note that if the two products are independent ($\gamma_1 = \gamma_2 = 0$), the service level S_i^{II} in (11) is identical to S_i^* in (7). It can be shown that for substitutable products, $S_i^{\mathrm{II}} > S_i^*$, whereas for complementary products, $S_i^{\mathrm{II}} < S_i^*$. (See Corollary 5 in §4.1.) Intuitively, both chains focus on inventory's net stimulating effect on own chain's demand and ignore the negative (positive) impact of its inventory on the substitutable (complementary) product, whereas the service level S_i^* in (7)

internalizes the inventory effect between chains.

From Theorem 2, an increases in the net stimulating effect $\alpha_i - \beta_i$ results in a higher profit for chain i, while it reduces (improves) the profit of the substitutable (complementary) product.

3.3 Two Decentralized Chains (DD)

Suppose that both chains are decentralized and, within each chain, the manufacturer and its retailers are coordinated via supply contracts. As discussed in §1, the focus of this paper is chain-to-chain competition, not competition between manufacturers. Thus, we assume the manufacturer and retailers are coordinated to maximize their own chain's profit. For many channel-coordinating contracts, the contract parameters can be adjusted to achieve various allocations of the supply chain profit between the manufacturer and the retailers.

The chain-to-chain competition in DD structure involves two stages. In the first stage, both chains decide their target service levels (via coordinating contracts). In the second stage, the four retailers compete in inventories under the given contracts.

In the first stage, various forms of contracts can serve the purpose of setting target service level. For example, under a buyback contract with wholesale price w_i and buyback rate b_i, retailer (i,j)'s problem becomes

$$\max_{q_{ij} \geq 0} (p_i - w_i)q_{ij} - (p_i - b_i)\mathsf{E}\big[(q_{ij} - D_{ij}(\boldsymbol{q}))^+\big]. \tag{14}$$

It can be verified that the above retailer's problem leads to the following service level:[1]

$$S_i^c = \frac{p_i - w_i}{(p_i - b_i)(1 - \alpha_i)}.$$

From this point onward, we do not assume any specific contract form. We let S_i^c denote the target service level chosen by chain i in the first stage. Then, using (5), the second-stage equilibrium inventory levels are

$$q_{i1} = q_{i2} = \frac{(1 - \alpha_{-i} + \beta_{-i})F_i^{-1}(S_i^c) - \gamma_{-i}F_{-i}^{-1}(S_{-i}^c)}{\eta}. \tag{15}$$

In the first-stage game, each chain anticipates the retailers' equilibrium decisions in (15) and

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[1] The standard newsvendor under buyback contract would aim at service level $(p_i - w_i)/(p_i - b_i)$. In our setting, the retailer takes into account inventory's stimulating effect on its own demand, and thus the service level becomes $(p_i - w_i)/[(p_i - b_i)(1 - \alpha_i)]$.

chooses service levels simultaneously. Chain i's problem can be written as (using (15)):

$$\max_{S_i^c} \ 2(p_i - c_i)\frac{(1 - \alpha_{-i} + \beta_{-i})F_i^{-1}(S_i^c) - \gamma_{-i}F_{-i}^{-1}(S_{-i}^c)}{\eta} - 2(p_i - v_i)\int_0^{F_i^{-1}(S_i^c)} F_i(u)du \qquad (16)$$

The objective in (16) is generally not concave, but the proof of Theorem 3 shows that this objective is quasi-concave in S_i^c. Hence, the optimal service level is determined by the first-order condition:

$$\frac{2(p_i - c_i)(1 - \alpha_{-i} + \beta_{-i})}{\eta}\frac{\partial F_i^{-1}(S_i^c)}{\partial S_i^c} - 2(p_i - v_i)S_i^c\frac{\partial F_i^{-1}(S_i^c)}{\partial S_i^c} = 0, \qquad (17)$$

which leads to the optimal service level given in the following theorem.

Theorem 3 (DD equilibrium) *When both chains are decentralized, the equilibrium service levels and inventory levels are given by $S_{ij} = S_i^{\mathrm{DD}}$ and $q_{ij} = q_i^{\mathrm{DD}}$, for $i, j = 1, 2$, where*

$$S_i^{\mathrm{DD}} \overset{\mathrm{def}}{=} \frac{p_i - c_i}{(p_i - v_i)(1 - \alpha_i + \beta_i - \frac{\gamma_1\gamma_2}{1 - \alpha_{-i} + \beta_{-i}})}, \qquad (18)$$

$$q_i^{\mathrm{DD}} \overset{\mathrm{def}}{=} \frac{1}{\eta}\Big[(1 - \alpha_{-i} + \beta_{-i})F_i^{-1}(S_i^{\mathrm{DD}}) - \gamma_{-i}F_{-i}^{-1}(S_{-i}^{\mathrm{DD}})\Big]. \qquad (19)$$

Chain i's profit in the equilibrium is

$$\pi_i^{\mathrm{DD}} = 2(p_i - c_i)q_i^{\mathrm{DD}} - 2(p_i - v_i)\int_0^{F_i^{-1}(S_i^{\mathrm{DD}})} F_i(u)du. \qquad (20)$$

Furthermore, S_i^{DD} increases in $\alpha_i - \beta_i$ and $\alpha_{-i} - \beta_{-i}$. If $\gamma < 0$, then q_i^{DD} and π_i^{DD} increase in $\alpha_i - \beta_i$ and $\alpha_{-i} - \beta_{-i}$.

In Theorems 1 and 2, q_i^* and q_i^{II} increase in $\alpha_i - \beta_i$, but in the DD structure when $\gamma > 0$, q_i^{DD} does not necessarily increase in $\alpha_i - \beta_i$. This is because the competing chain's service level S_{-i}^{DD} increases in $\alpha_i - \beta_i$, which may cause q_i^{DD} in (19) to decrease, and π_i^{DD} may decline as well. When $\gamma < 0$, as $\alpha_i - \beta_i$ increases, inventory levels and profits of both chains increase.

It is important to note that the coordinated decentralized chains do not behave as the integrated chains in the II structure. Integrated chains compete directly using inventory levels: In the II equilibrium, chain i's inventory levels are optimized given the other chain's inventory decisions. However, decentralized chains do not decide inventory directly, but decide service levels in the first stage. Both chains recognize that their own service levels will influence the competing chain's inventory levels in the second stage, reflected by (5) and elaborated below:

- If the two products are substitutes, then a high own chain's service level reduces the competing chain's inventory. This effect provides incentives for both decentralized chains to raise their service levels above those in the II equilibrium.

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- If the two products are complements, then a high own chain's service level increases the competing chain's inventory, which in turn stimulate own chain's demand. This effect also provides incentives for both chains to raise their service levels above those in the II equilibrium.

Summarizing the above discussion, the service level S_i^{DD} in (18) is higher than S_i^{II} in (11), regardless of the whether the products are substitutes or complements. The only situation that $S_i^{\mathrm{DD}} = S_i^{\mathrm{II}}$ is when the two products are independent ($\gamma = 0$).

3.4 A Decentralized Chain Competing with an Integrated Chain (DI)

Suppose that chain 1 is decentralized and chain 2 is integrated. In the decentralized chain, the manufacturer and the retailers are coordinated via contracts. Unlike the DD structure where neither chain can decide inventory levels in the first-stage game, in the DI structure, the integrated chain decides inventory levels, while the decentralized chain decides its service level. In the second stage, the decentralized retailers decide inventory levels.

We first discuss the intuitions on the equilibrium behavior, and then formally prove these intuitions. Focus on the decentralized chain (chain 1) first. If the competing chain (chain 2) is also decentralized, the analysis on DD structure shows that chain 1 chooses service level to influence the chain 2's inventory decisions in the second stage. If the chain 2 is integrated, however, it commits inventory levels in the first stage, and thus, the best response of chain 1 is to induce its retailers to choose the best inventory levels in response to chain 2's inventory, which coincides with the best response problem of an integrated chain in the II structure. Therefore, chain 1's behavior may be similar to an integrated chain in the II structure even though it is decentralized.

We now turn to the integrated chain 2 in the DI structure. Chain 2 understands that it makes inventory decisions earlier than chain 1's retailers decide inventory. Chain 2 also recognizes that its service level influences chain 1's inventory decision, reflected by (5). Hence, chain 2's problem is similar to that in the DD structure. Therefore, chain 2's behavior may be similar to a decentralized chain in the DD structure even though it is integrated.

We now formalize the above intuitions. In the first-stage game, chain 1 chooses coordinating contract and chain 2 decides inventory. Consider chain 1's best response problem to chain 2's inventory q_{21} and q_{22}. Chain 1 aims to maximize its profit by choosing a contract and its parameters. This contract should induce chain 1's retailers to choose the best inventory levels in response to chain 2's inventory levels. From the II structure in Theorem 2, we know that the service level that

maximizes chain 1's profit is given by S_1^{II} in (11). Hence, chain 1 will set a contract such that

$$S_{1j} = S_1^{\mathrm{II}}. \tag{21}$$

This confirms our intuition that the behavior of the decentralized chain in the DI structure is similar to an integrated chain in the II structure.

Using (3), we can write the condition (21) as

$$S_{1j} = F_1\Big((1-\alpha_1))q_{1j} + \beta_1 q_{1,-j} + \frac{1}{2}\gamma_2(q_{21} + q_{22})\Big) = S_1^{\mathrm{II}}, \qquad j = 1, 2.$$

From the above equation, we can find chain 1's inventory level as a best response to chain 2's inventory $q_{21} + q_{22}$:

$$q_{11} + q_{12} = \frac{-\gamma_2(q_{21} + q_{22}) + 2F_1^{-1}(S_1^{\mathrm{II}})}{1 - \alpha_1 + \beta_1}. \tag{22}$$

Clearly, for substitutable (complementary) products, inventories are also strategic substitutes (complements).

Next, we consider the integrated chain 2 making inventory decisions in anticipation of chain 1's response in (22). Using (22), chain 2's problem can be written as

$$\max_{q_{21}, q_{22}} \sum_j \Big[(p_2 - c_2)q_{2j} - (p_2 - v_2)\mathsf{E}\big[\big(q_{2j} - D_{2j}(q_{12}, q_{22})\big)^+\big]\Big], \tag{23}$$

where

$$D_{2j}(q_{21}, q_{22}) \stackrel{\text{def}}{=} \xi_{2j} + \alpha_2 q_{2j} - \beta_2 q_{2,-j} + \frac{\gamma_1\gamma_2(q_{21} + q_{22})}{2(1 - \alpha_1 + \beta_1)} - \frac{\gamma_1 F_1^{-1}(S_1^{\mathrm{II}})}{1 - \alpha_1 + \beta_1}. \tag{24}$$

Because $D_{2j}(q_{21}, q_{22})$ is linear in q_{21} and q_{22}, the objective function in (23) is concave in q_{21} and q_{22}.

Using (24), the objective in (23) can be written as

$$\sum_j \Big[(p_2 - c_2)q_{2j} - (p_2 - v_2) \int_0^{q_{2j}} F_2\Big(u - \alpha_2 q_{2j} + \beta_2 q_{2,-j} - \frac{\gamma_1\gamma_2(q_{21} + q_{22})}{2(1 - \alpha_1 + \beta_1)} + \frac{\gamma_1 F_1^{-1}(S_1^{\mathrm{II}})}{1 - \alpha_1 + \beta_1}\Big)du\Big].$$

The first-order condition is

$$\frac{p_2 - c_2}{p_2 - v_2} = \Big(1 - \alpha_2 - \frac{\gamma_1\gamma_2}{2(1 - \alpha_1 + \beta_1)}\Big)S_{2j} + \Big(\beta_2 - \frac{\gamma_1\gamma_2}{2(1 - \alpha_1 + \beta_1)}\Big)S_{2,-j}, \qquad j = 1, 2.$$

The above optimality condition is equivalent to

$$S_{2j} = S_2^{\mathrm{DD}}, \qquad j = 1, 2,$$

where S_2^{DD} is defined in (18). This proves our intuition that the behavior of the integrated chain in the DI structure is similar to a decentralized chain in the DD structure.

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The above results are summarized in the following theorem.

Theorem 4 (DI structure) *When chain 1 is decentralized and chain 2 is integrated, the equilibrium service level and inventory level are given by $S_{ij} = S_i^{\text{DI}}$ and $q_{ij} = q_i^{\text{DI}}$, for $i, j = 1, 2$, where*

$$S_1^{\text{DI}} = S_1^{\text{II}} = \frac{p_1 - c_1}{(p_1 - v_1)(1 - \alpha_1 + \beta_1)}, \tag{25}$$

$$S_2^{\text{DI}} = S_2^{\text{DD}} = \frac{p_2 - c_2}{(p_2 - v_2)(1 - \alpha_2 + \beta_2 - \frac{\gamma_1 \gamma_2}{1 - \alpha_1 + \beta_1})}, \tag{26}$$

$$q_i^{\text{DI}} \overset{\text{def}}{=} \frac{1}{\eta} \left[(1 - \alpha_{-i} + \beta_{-i}) F_i^{-1}(S_i^{\text{DI}}) - \gamma_{-i} F_{-i}^{-1}(S_{-i}^{\text{DI}}) \right]. \tag{27}$$

Chain i's profit in the equilibrium is

$$\pi_i^{\text{DI}} = 2(p_i - c_i) q_i^{\text{DI}} - 2(p_i - v_i) \int_0^{F_i^{-1}(S_i^{\text{DI}})} F_i(u) du. \tag{28}$$

Furthermore, S_2^{DI}, q_2^{DI}, and π_2^{II} increase in $\alpha_2 - \beta_2$; q_1^{DI} and π_1^{DI} decrease (increase) in $\alpha_2 - \beta_2$ if $\gamma > 0$ ($\gamma < 0$); S_1^{DI} and S_2^{DI} increase in $\alpha_1 - \beta_1$; q_1^{DI} and π_1^{DI} increase in $\alpha_1 - \beta_1$ if $\gamma < 0$.

The effects of integrated chain's $\alpha_2 - \beta_2$ on the DI equilibrium are qualitatively the same as the II equilibrium in Theorem 2, because $\alpha_2 - \beta_2$ affects the service levels in the same way as in the II equilibrium. On the other hand, an increase in $\alpha_1 - \beta_1$ raises both S_1^{DI} and S_2^{DI}, and thus, the effects of $\alpha_1 - \beta_1$ on the DI equilibrium are qualitatively the same as the DD equilibrium in Theorem 3.

4. Equilibrium Supply Chain Structures

The previous section analyzes the firms' strategic interactions under any given supply chain structure. In this section, we consider supply chain structure choice game, in which both chains simultaneously choose to be either integrated or decentralized. We identify the equilibrium supply chain structure and further analyze the impact of demand uncertainty on the equilibrium choice.

4.1 Comparison of Industry Structures

With the equilibria solved in §§3.2-3.4, we compare the equilibrium service levels, inventory levels, and profits in this section.

Corollary 5 *If $\gamma > 0$, then $S_i^* < S_i^{\text{II}} < S_i^{\text{DD}}$;   if $\gamma < 0$, then $S_i^{\text{II}} < S_i^{\text{DD}} < S_i^*$.*

The inequality $S_i^{\text{II}} < S_i^{\text{DD}}$ means that the decentralized chains raise service levels to either reduce the demand-stealing effect of competing chain's inventory or increase the demand-stimulating effect

of competing chain's inventory. These are discussed in detail after Theorem 3.

For substitutable products, chain-to-chain competition results in a higher service levels than if the entire industry is integrated. For complementary products, each supply chain enjoys the demand-stimulating effect from the other chain's inventory and thus offers lower service level than if the industry is integrated.

Next, we compare the total industry profit. Let X denote a supply chain structure, $X \in \{*, II, DD, DI\}$, where "$*$" denote the integrated industry structure. Let π^X denote the expected profit of the industry under structure X, which can be written as

$$
\begin{aligned}
\pi^X &= \pi_1^X + \pi_2^X \\
&= \sum_i \left[2(p_i - c_i)q_i^X - 2(p_i - v_i) \int_0^{F_i^{-1}(S_i^X)} F_i(u)du \right] \\
&= \sum_i \left[\frac{2(p_i - c_i)}{\eta} \left[(1 - \alpha_{-i} + \beta_{-i})F_i^{-1}(S_i^X) - \gamma_{-i}F_{-i}^{-1}(S_{-i}^X) \right] - 2(p_i - v_i) \int_0^{F_i^{-1}(S_i^X)} F_i(u)du \right] \\
&= \sum_i \left[\frac{2}{\eta} \left[(p_i - c_i)(1 - \alpha_{-i} + \beta_{-i}) - \gamma_i(p_{-i} - c_{-i}) \right] F_i^{-1}(S_i^X) - 2(p_i - v_i) \int_0^{F_i^{-1}(S_i^X)} F_i(u)du \right]
\end{aligned}
$$

Note that S_i^* in (7) can be written as

$$
S_i^* = \frac{(p_i - c_i)(1 - \alpha_{-i} + \beta_{-i}) - \gamma_i(p_{-i} - c_{-i})}{(p_i - v_i)\eta}.
$$

Hence, we obtain an expression for the total industry profit:

$$
\pi^X = \sum_i 2(p_i - v_i) \, g_i(S_i^X), \tag{29}
$$

where

$$
g_i(S) \overset{\text{def}}{=} S_i^* F_i^{-1}(S) - \int_0^{F_i^{-1}(S)} F_i(u)du. \tag{30}
$$

The slope of $g_i(S)$ is

$$
\frac{dg_i(S)}{dS} = (S_i^* - S)\frac{dF_i^{-1}(S)}{dS}. \tag{31}
$$

Using Corollary 5 and (31), it can be seen that when $\gamma > 0$, $g_i(S)$ strictly decreases in S for $S \in [S_i^*, S_i^{DD}]$, and thus, $g_i(S_i^{II}) > g_i(S_i^{DD})$. Therefore, (29) implies that $\pi^{DD} < \pi^{DI} < \pi^{II}$. On the other hand, when $\gamma < 0$, $g_i(S)$ strictly increases in S for $S \in [S_i^{II}, S_i^*]$, and thus, $g_i(S_i^{II}) < g_i(S_i^{DD})$. Therefore, (29) implies that $\pi^{II} < \pi^{DI} < \pi^{DD}$.

The above results are summarized in the following corollary.

Corollary 6 *If $\gamma > 0$, then $\pi^{\mathrm{DD}} < \pi^{\mathrm{DI}} < \pi^{\mathrm{II}} < \pi^*$. If $\gamma < 0$, then $\pi^{\mathrm{II}} < \pi^{\mathrm{DI}} < \pi^{\mathrm{DD}} < \pi^*$.*

The order of profits in Corollary 6 is consistent with the order of service levels in Corollary 5 in the following sense: The more the industry service level deviates from the industry optimal service level, the lower the total industry profit.

The above two corollaries compare the industry-wide service levels and profits. Next, we compare supply chains' inventory levels and profits.

Corollary 7 *If two products are complements ($\gamma < 0$),*

$$q_i^{\mathrm{II}} \; < \; q_i^{\mathrm{DI}} \; < \; q_i^{\mathrm{DD}} \; < \; q_i^*,$$

$$\pi_i^{\mathrm{II}} \; < \; \pi_i^{\mathrm{DI}} \; < \; \pi_i^{\mathrm{DD}} \; < \; \pi_i^*.$$

If two products are substitutes ($\gamma > 0$),

$$q_1^{\mathrm{DI}} < \min\{q_1^{\mathrm{II}}, q_1^{\mathrm{DD}}\}, \qquad q_2^{\mathrm{DI}} > \max\{q_2^{\mathrm{II}}, q_2^{\mathrm{DD}}\},$$

$$\pi_1^{\mathrm{DI}} < \min\{\pi_1^{\mathrm{II}}, \pi_1^{\mathrm{DD}}\}, \qquad \pi_2^{\mathrm{DI}} > \max\{\pi_2^{\mathrm{II}}, \pi_2^{\mathrm{DD}}\}.$$

When $\gamma > 0$, the comparison between q_i^{II}, q_i^{DD}, and q_i^* depends on the demand distribution. To see this dependence, note that $q_i^{\mathrm{X}} = \frac{1}{\eta}\Big[(1 - \alpha_{-i} + \beta_{-i})F_i^{-1}(S_i^{\mathrm{X}}) - \gamma_{-i}F_{-i}^{-1}(S_{-i}^{\mathrm{X}})\Big]$, for $\mathrm{X} \in \{*, \mathrm{II}, \mathrm{DD}\}$. Because $\gamma > 0$ and $S_i^* < S_i^{\mathrm{II}} < S_i^{\mathrm{DD}}$, the order of q_i^{II}, q_i^{DD}, and q_i^* depends on the shape of $F_i^{-1}(S)$ and $F_{-i}^{-1}(S)$ for $S \in [S_i^*, S_i^{\mathrm{DD}}]$. Similar indeterminacy occurs for ordering the profit under $\mathrm{X} \in \{*, \mathrm{II}, \mathrm{DD}\}$.

Corollary 7 reveals that, for complementary products, the individual chain's inventory and profit under various structures have the same order as the industry-wide inventory and profit. However, for substitutable products, although the industry's profits are ordered as $\pi^{\mathrm{DD}} < \pi^{\mathrm{DI}} < \pi^{\mathrm{II}}$ (see Corollary 6), individual supply chains' profits do not have the same order. The integrated chain in the DI structure has the highest inventory and profit, whereas the decentralized chain in the DI structure has the lowest inventory and profit.

4.2 Equilibrium Supply Chain Structures

We now consider a supply chain structure game, in which both supply chains simultaneously choose to be integrated or decentralized, in anticipation of the profits given in the previous sections.

Consider substitutable products ($\gamma > 0$) first. Corollary 7 shows that $\pi_1^{\mathrm{II}} > \pi_1^{\mathrm{DI}}$, i.e., if chain 2 is integrated, then chain 1 prefers integration as well. By symmetry, if chain 1 is integrated, then chain 2 also prefers integration. These together imply that II structure is a Nash equilibrium supply

chain structure.

When $\gamma > 0$, Corollary 7 also shows that $\pi_2^{DD} < \pi_2^{DI}$, i.e., if chain 1 is decentralized, then chain 2 prefers integration, which asserts that integration is a dominant strategy for each supply chain when the two products are substitutable.

Next, consider the case of complementary products ($\gamma < 0$). Corollary 7 shows that $\pi_i^{DI} < \pi_i^{DD}$, i.e., if one chain is decentralized, then the other chain prefers to be decentralized. Furthermore, $\pi_i^{II} < \pi_i^{DI}$ implies that if one chain is integrated, then the other chain still prefers decentralization. This reveals that decentralization is a dominant strategy for each supply chain when the two products are complementary.

The above discussions are summarized in the following theorem.

Theorem 8 *If two products are substitutable,* II *is the unique equilibrium supply chain structure. If two products are complementary,* DD *is the unique equilibrium supply chain structure.*

Corollary 6 implies that for substitutable (complementary) products, the II (DD) structure brings the industry the highest profit among all duopoly structures, and Theorem 8 further confirms that the II (DD) structure is the industry's only equilibrium structure.

The reasons for II and DD to be the equilibria under different product characteristics are explained below.

Note that when a chain is decentralized, its inventory decision is delayed compared to an integrated chain, because the chain members need to negotiate on the contract terms in the first stage. This is why in the DI structure, the decentralized retailers choose their inventories after the integrated chain decides inventory levels. In fact, the DI equilibrium coincides with the equilibrium of a Stakelberg game, in which one chain chooses inventory first, and the other chain follows.

For substitutable products ($\gamma > 0$), inventories are strategic substitutes (see equation (22)) and the first-mover advantage emerges. Therefore, both chains want to be the leader by committing inventory levels as early as possible, resulting in II being the equilibrium industry structure. Neither chain has an incentive to decentralize because a delayed inventory decision will bring disadvantage to the chain when products are substituable.

For complementary products ($\gamma < 0$), inventories are strategic complements and the first-mover advantage disappears. Both supply chains want to be the followers, resulting in DD being the equilibrium. Neither chain has an incentive to integrate because with two complementary products, early inventory decision only brings disadvantage to the integrated chain.

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4.3 Impact of Demand Uncertainty

Finally, we explore the impact of demand uncertainty on supply chain's profits. A common definition of variability order can be found in Ross (1996). Let X and Y be random variables. X is said to be more variable than Y, denoted by $X \geq_v Y$, if $\mathsf{E}[h(X)] \geq \mathsf{E}[h(X)]$, for all increasing convex h. If X and Y are nonnegative random variables with distributions F and G respectively, then $X \geq_v Y$ if and only if $\int_a^\infty (G(x) - F(x))dx \geq 0$, for all $a \geq 0$.

In this paper, we use a different stochastic ordering, which we call dispersion. For simplicity, we focus on the case where distributions and inverse distributions are all continuous functions.

Definition 1 *Let X and Y be random variables with cumulative distributions F and G respectively, and suppose that F, G, F^{-1} and G^{-1} are continuous. X is said to be more dispersed than Y, denoted by $X \geq_d Y$ or $F \geq_d G$, if*

$$F^{-1}(b) - F^{-1}(a) \geq G^{-1}(b) - G^{-1}(a), \qquad \text{for all } 0 < a < b < 1. \tag{32}$$

If the inverse distributions are differentiable, then Definition 1 is equivalent to

$$\frac{dF^{-1}(y)}{dy} \geq \frac{dG^{-1}(y)}{dy}, \qquad \text{for all } y \in (0,1). \tag{33}$$

That is, X is more dispersed than Y if F^{-1} is steeper than G^{-1} (or F is flatter than G) everywhere. It can be shown that $\geq_d$ is a complete order for most of commonly used class of distributions; and that $\geq_d$ coupled with $\mathsf{E}[X] \geq \mathsf{E}[Y]$ implies $\geq_v$, as stated below.

Lemma 9 *If $X \geq_d Y$, then*

(a) *If $F(\widehat{x}) = G(\widehat{x}) = \widehat{y} \in (0,1)$ for some $\widehat{x}$, then $F(x) \geq G(x)$ for all $x \leq \widehat{x}$ and $F(x) \leq G(x)$ for all $x \geq \widehat{x}$; $F^{-1}(y) \leq G^{-1}(y)$, for all $0 < y \leq \widehat{y}$, and $F^{-1}(y) \geq G^{-1}(y)$, for all $\widehat{y} \leq y < 1$.*

(b) *If X and Y are non-negative, and $\mathsf{E}[X] \geq \mathsf{E}[Y]$, then $X \geq_v Y$;*

(c) *For $y \in (0,1)$, $\int_0^{F^{-1}(y)} F(u)du \geq \int_0^{G^{-1}(y)} G(u)du$, and $\int_{F^{-1}(y)}^\infty (1-F(u))du \geq \int_{G^{-1}(y)}^\infty (1-G(u))du$.*

To emphasize the dependence on demand uncertainty, we rewrite (29) and (30) as

$$\pi^X(F_1, F_2) = \sum_i 2(p_i - v_i)g_i(S_i^X; F_i),$$

$$g_i(y; F) = S_i^* F^{-1}(y) - \int_0^{F^{-1}(y)} F(u)du.$$

Now consider two demand systems of the form specified in (1). All parameters are the same and the mean demands are also the same, except that one system is more dispersed than the other.

Specifically, the two demand systems are labeled by superscript F and G; we assume $\xi_{ij}^F \sim F_i$, $\xi_{ij}^G \sim G_i$, and that $\mathsf{E}[\xi_{ij}^F] = \mathsf{E}[\xi_{ij}^G]$, for $i, j = 1, 2$.

The following theorem addresses the impact of demand dispersion on the expected supply chain profit.

Theorem 10 *If $F_i \geq_d G_i$, $i = 1, 2$, then for any structure X, $\pi^X(F_1, F_2) \leq \pi^X(G_1, G_2)$. Furthermore, if $F_i \geq_d G_i$ and $F_{-i} = G_{-i}$, then for any structure X, $\pi_i^X(F_1, F_2) \leq \pi_i^X(G_1, G_2)$.*

We essentially prove that $g_i(y, F)$ decreases when F becomes more dispersed, by exploring the properties of dispersion order stated in Lemma 9. The theorem shows that the expected industry profit is lower under larger demand dispersion (with mean demand kept constant). The theorem also shows that a high own-demand dispersion reduces own chain's profit. However, the impact of the other chain's demand dispersion on own profit is ambiguous. It depends on how the dispersion is shaped.

Finally, we explore the impact of demand uncertainty on the relative profit when comparing two different structures.

Theorem 11 *If $F_i \geq_d G_i$ for $i = 1, 2$, then for any two structures X_1 and X_2,*

$$\left| \pi^{X_1}(F_1, F_2) - \pi^{X_2}(F_1, F_2) \right| \geq \left| \pi^{X_1}(G_1, G_2) - \pi^{X_2}(G_1, G_2) \right|. \tag{34}$$

For individual supply chain, when $\gamma > 0$, $F_2 \geq_d G_2$ and $F_1 = G_1$,

$$\pi_1^{II}(F_1, F_2) - \pi_1^{DI}(F_1, F_2) \geq \pi_1^{II}(G_1, G_2) - \pi_1^{DI}(G_1, G_2); \tag{35}$$

and when $\gamma < 0$, $F_1 \geq_d G_1$ and $F_2 = G_2$,

$$\pi_2^{DD}(F_1, F_2) - \pi_2^{DI}(F_1, F_2) \geq \pi_2^{DD}(G_1, G_2) - \pi_2^{DI}(G_1, G_2). \tag{36}$$

The first part of the theorem shows that the profit differences of different industry structures are increasing as the demand becomes more uncertain.

Inequality (35) shows that for substitutable products, if the other chain is integrated, the advantage of integration over decentralization to own chain increases as the demand of *the other* chain becomes more dispersed. Inequality (36) shows that for complementary products, if the other chain is decentralized, the advantage of decentralization over integration to own chain increases as the demand of *the other* chain becomes more uncertain.

In summary, a high demand uncertainty reinforces the equilibrium supply chain structure stated

in Theorem 8.

5. Conclusions

In this paper, we link the analytical marketing literature on channel structures to the operations management literature on inventory decisions under demand uncertainty. We examine the relations between supply chain structures, supply chain performance, product substitutability/complementarity, and demand uncertainty.

We consider three duopoly structures: two integrated chains (II), two decentralized chains (DD), and a mixed structure with one decentralized chain and one integrated chain (DI). We derive firms' equilibrium strategies in explicit forms, which allow us to compare different supply chain structures and further analyze the equilibrium structure choice. We assume fixed prices and focus on inventory competition, in which inventory levels impact retailers' demands through both stealing and stimulating effects.

We find that the DD structure always induces higher service levels than the II structure, regardless whether the products are substitutes or complements. This is because in the DD structure, committing to a higher service level in the first stage can reduce (increase) the competing chain's inventory of substitutable (complementary) products in the second stage, which incentivizes both decentralized chains to raise their service levels higher than in the II structure. In the DI structure, decentralization delays a chain's inventory decision, allowing the integrated chain to decide inventory first. As a result, the decentralized chain's best service level is the same as a follower in the inventory competition.

In terms of supply chain profits, we find that for substitutable products, the integrated chain (leading in inventory decision) in DI structure earns more profit than it would earn in the II and DD structures, while the decentralized chain in the DI structure earns less than in the other two structures. Therefore, integration is a dominant strategy for each supply chain when the two products are substitutable. The equilibrium structure choice is II, which is also the most profitable duopoly structure (the industry profits are ordered as II > DI > DD). On the other hand, when the products are complements, the industry profits are ordered as DD > DI > II, and the individual supply chain profit has the same order. Hence, decentralization is a dominant strategy for each supply chain. The equilibrium structure choice is DD, which is also the most profitable duopoly structure.

Finally, we find that when demands become more uncertain, the cost of deviating from the

equilibrium supply chain structure is larger. Thus, demand uncertainty reinforces the equilibrium supply chain structure.

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Appendix: Proofs

Proof of Theorem 1. The expected excess inventory at retailer (i,j) can be expressed as

$$\mathsf{E}[(q_{ij} - D_{ij}(\boldsymbol{q}))^+] = \int_0^{q_{ij}} \mathsf{P}[D_{ij}(\boldsymbol{q}) \leq u]du$$

$$= \int_0^{q_{ij}} F_i\Big(u - \alpha_i q_{ij} + \beta_i q_{i,-j} + \frac{1}{2}\gamma_{-i}(q_{-i,1} + q_{-i,2})\Big)\,du$$

$$= \int_0^{F_i^{-1}(S_{ij})} F_i(u)du. \tag{A.1}$$

The industry's profit function in (6) can be written as

$$\sum_{i,j} \left[(p_i - c_i)q_{ij} - (p_i - v_i)\int_0^{q_{ij}} F_i\Big(u - \alpha_i q_{ij} + \beta_i q_{i,-j} + \frac{1}{2}\gamma_{-i}(q_{-i,1} + q_{-i,2})\Big)du\right].$$

The discussion after (6) reveals the concavity of the objective function. Hence, the optimal inventory levels are given by the first-order conditions:

$$p_i - c_i = (p_i - v_i)\big((1 - \alpha_i)S_{ij} + \beta_i S_{i,-j}\big) + \frac{1}{2}(p_{-i} - v_{-i})\gamma_i(S_{-i,j} + S_{-i,-j}), \quad i,j = 1,2,$$

where we used the service level defined in (3). This system of equations determines the optimal service levels in (7), which in turn determine the optimal inventory in (8) by using the relation in (5). The optimal inventory and service level and the expression in (A.1) lead to the profit in (9).

Next, we prove the monotonicity properties. The optimal service level in (7) can be written as

$$S_i^* = \frac{(1 - \alpha_{-i} + \beta_{-i})(p_i - c_i) - \gamma_i(p_{-i} - c_{-i})}{(p_i - v_i)((1 - \alpha_1 + \beta_1)(1 - \alpha_2 + \beta_2) - \gamma_1\gamma_2)} = \frac{(1 - \alpha_{-i} + \beta_{-i})(p_i - c_i) - \gamma_i(p_{-i} - c_{-i})}{(p_i - v_i)\eta}.$$

We can derive

$$\frac{\partial S_i^*}{\partial(\alpha_i - \beta_i)} \geq 0,$$

$$\frac{\partial S_{-i}^*}{\partial(\alpha_i - \beta_i)} = \frac{(p_{-i} - c_{-i})\gamma_1\gamma_2 - (1 - \alpha_{-i} + \beta_{-i})\gamma_{-i}(p_i - c_i)}{(p_{-i} - v_{-i})\eta^2} = \frac{-\gamma_{-i}(p_i - v_i)\gamma_i}{(p_{-i} - v_{-i})\eta}.$$

Thus S_{-i}^* decreases (increases) in $\alpha_i - \beta_i$ if $\gamma > 0$ (< 0). Consequently, $\gamma_{-i}F_{-i}^{-1}(S_{-i}^*)$ decreases in $\alpha_i - \beta_i$.

Therefore, in equation (8), when $\alpha_i - \beta_i$ increases, the denominator decreases while the numerator increases. Thus q_i^* increases in $\alpha_i - \beta_i$.

To examine the relation between q_i^* and $\alpha_{-i} - \beta_{-i}$, we derive

$$\frac{\partial q_i^*}{\partial(\alpha_{-i} - \beta_{-i})} = -\frac{\gamma_{-i}}{\eta}\left[q_{-i}^* - \frac{1 - \alpha_{-i} + \beta_{-i}}{\gamma_{-i}}\frac{\partial F_i^{-1}}{\partial S_i}\frac{\partial S_i^*}{\partial(\alpha_{-i} - \beta_{-i})} + \frac{\partial F_{-i}^{-1}}{\partial S_{-i}}\frac{\partial S_{-i}}{\partial(\alpha_{-i} - \beta_{-i})}\right]. \tag{A.2}$$

All three terms in the brackets are positive, and thus, the sign of the above derivative only depends on the sign of γ in front of the brackets.

In equilibrium,

$$D_{ij}(\boldsymbol{q}^*) = \xi_{ij} + (\alpha_i - \beta_i)q_i^* - \gamma_{-i}q_{-i}^*.$$

Suppose $\alpha_i - \beta_i$ increases or γ_i decreases, and suppose the industry keeps the inventory levels $\boldsymbol{q}^*$ unchanged. Then, the above relation suggests that demand D_{ij} will increase, and the industry's profit increases. Therefore, if the industry optimally chooses inventory levels in response to the changes in $\alpha_i - \beta_i$ and γ_i, π^* will increase further.

Proof of Theorem 2. The problem in (10) can be written as

$$\max_{q_{i1},q_{i2}} \sum_j \left[(p_i - c_i)q_{ij} - (p_i - v_i)\int_0^{q_{ij}} F_i\Big(u - \alpha_i q_{ij} + \beta_i q_{i,-j} + \frac{1}{2}\gamma_{-i}(q_{-i,1} + q_{-i,2})\Big) du \right].$$

The first-order condition is

$$p_i - c_i = (p_i - v_i)\big((1 - \alpha_i)S_{ij} + \beta_i S_{i,-j}\big), \qquad j = 1, 2.$$

Solving the this system of equations, we obtain the optimal service level in (11).

Using (5), the equilibrium inventory levels are given by (12). The expression for chain i's profit in (13) can be derived in the same way as in the proof of Theorem 1.

We next prove the monotonicity. If $\alpha_i - \beta_i$ increases, then S_i^{II} in (11) increases, which also increases $F_i^{-1}(S_i^{\mathrm{II}})$ in (12). At the same time, a higher $\alpha_i - \beta_i$ reduces η. Hence, q_i^{II} increases in $\alpha_i - \beta_i$.

Differentiating q_i^{II} with respect to $\alpha_{-i} - \beta_{-i}$ and after some algebraic manipulations, we obtain

$$\frac{\partial q_i^{\mathrm{II}}}{\partial(\alpha_{-i} - \beta_{-i})} = -\frac{\gamma_{-i}}{\eta}\left[\frac{\partial F_{-i}^{-1}}{\partial S_{-i}}\frac{\partial S_{-i}^{\mathrm{II}}}{\partial(\alpha_{-i} - \beta_{-i})} + q_{-i}^{\mathrm{II}}\right]. \tag{A.3}$$

All terms in the square bracket are positive, and therefore, if $\gamma > 0$ ($\gamma < 0$), q_i^{II} decreases (increases) in $\alpha_{-i} - \beta_{-i}$.

From the profit expression in (13), we see that $\alpha_{-i} - \beta_{-i}$ affects π_i^{II} only through q_i^{II}. Thus, if $\gamma > 0$ ($\gamma < 0$), π_i^{II} decreases (increases) in $\alpha_{-i} - \beta_{-i}$.

Finally, we prove the monotonicity of π_i^{II} in $\alpha_i - \beta_i$. Let $\alpha_i - \beta_i < \widetilde{\alpha}_i - \widetilde{\beta}_i$. Under $\alpha_i - \beta_i$, using (3), the equilibrium satisfies

$$F_i\big((1 - \alpha_i + \beta_i)q_i^{\mathrm{II}} + \gamma_{-i}q_{-i}^{\mathrm{II}}\big) = S_i^{\mathrm{II}}.$$

Under $\widetilde{\alpha}_i - \widetilde{\beta}_i$, denote the new equilibrium inventory as $\widetilde{q}_1^{\mathrm{II}}$ and $\widetilde{q}_2^{\mathrm{II}}$. Suppose chain $-i$ does choose the equilibrium inventory $\widetilde{q}_{-i}^{\mathrm{II}}$, but chain i takes a suboptimal strategy q_i^s to maintain its original

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service level under $\alpha_i - \beta_i$. That is,

$$F_i\big((1 - \widetilde{\alpha}_i + \widetilde{\beta}_i)q_i^s + \gamma_{-i}\widetilde{q}_{-i}^{\mathrm{II}}\big) = S_i^{\mathrm{II}}.$$

As we show by (A.3), if $\gamma > 0$ ($\gamma < 0$), q_{-i}^{II} decreases (increases) in $\alpha_i - \beta_i$. In either case, $\gamma q_{-i}^{\mathrm{II}}$ decreases in $\alpha_i - \beta_i$. Hence, $\gamma_{-i}q_{-i}^{\mathrm{II}} > \gamma_{-i}\widetilde{q}_{-i}^{\mathrm{II}}$. Together with $1 - \alpha_i + \beta_i > 1 - \widetilde{\alpha}_i + \widetilde{\beta}_i > 0$, we have

$$q_i^s > q_i^{\mathrm{II}}.$$

Under this suboptimal inventory level q_i^s, the profit of chain i is

$$
\begin{aligned}
\pi_i &= 2(p_i - c_i)q_i^s - 2(p_i - v_i)\int_0^{F_i^{-1}(S_i^{\mathrm{II}})} F_i(u)du \\
&> 2(p_i - c_i)q_i^{\mathrm{II}} - 2(p_i - v_i)\int_0^{F_i^{-1}(S_i^{\mathrm{II}})} F_i(u)du \ = \ \pi_i^{\mathrm{II}}.
\end{aligned}
$$

Since a suboptimal inventory for chain i yields a higher profit, the optimal profit π_i^{II} in the equilibrium must be even higher. The proves π_i^{II} increases in $\alpha_i - \beta_i$.

Proof of Theorem 3. To show the quasi-concavity of the objective function in (16), it suffices to show that the first-order derivative crosses zero value from above only once. The first-order derivative with respect to S_i^c is

$$\left[\frac{2(p_i - c_i)(1 - \alpha_{-i} + \beta_{-i})}{\eta} - 2(p_i - v_i)S_i^c\right]\frac{\partial F_i^{-1}(S_i^c)}{\partial S_i^c}. \tag{A.4}$$

Because $F_i^{-1}(y)$ is assumed to be strictly increasing in y for $y \in (0,1)$, we have $\partial F_i^{-1}/\partial S_i^c > 0$. The term in the brackets is strictly decreasing in S_i^c. Hence, as S_i^c increases, the derivative in (A.4) crosses zero value from above only once. Thus, the objective function in (16) is quasi-concave.

Service level S_i^{DD} in (18) clearly increases in $\alpha_1 - \beta_1$ and $\alpha_2 - \beta_2$.

If $\gamma < 0$, then q_i^{DD} in (19) increases in $\alpha_i - \beta_i$, because as $\alpha_i - \beta_i$ increases, η decreases, $F_i^{-1}(S_i^{\mathrm{DD}})$ increases, and $-\gamma_{-i}F_{-i}^{-1}(S_{-i}^{\mathrm{DD}})$ increases.

Similar to (A.2), we can derive

$$\frac{\partial q_i^{\mathrm{DD}}}{\partial(\alpha_{-i} - \beta_{-i})} = -\frac{\gamma_{-i}}{\eta}\left[q_{-i}^{\mathrm{DD}} + \frac{\partial F_{-i}^{-1}}{\partial S_{-i}}\frac{\partial S_{-i}}{\partial(\alpha_{-i} - \beta_{-i})}\right] + \frac{1 - \alpha_{-i} + \beta_{-i}}{\eta}\frac{\partial F_i^{-1}}{\partial S_i}\frac{\partial S_i^{\mathrm{DD}}}{\partial(\alpha_{-i} - \beta_{-i})}$$

Because $\gamma < 0$ and S_i^{DD} increases in $\alpha_{-i} - \beta_{-i}$, the above derivative is positive, and thus q_i^{DD} increases in $\alpha_{-i} - \beta_{-i}$.

Finally, we prove the monotonicity of π_i^{DD} in $\alpha_i - \beta_i$ using the same method as in the proof of the monotonicity of π_i^{II} in $\alpha_i - \beta_i$ in Theorem 2. Let $\alpha_i - \beta_i < \widetilde{\alpha}_i - \widetilde{\beta}_i$. Under $\alpha_i - \beta_i$, the equilibrium satisfies $F_i\big((1 - \alpha_i + \beta_i)q_i^{\mathrm{DD}} + \gamma_{-i}q_{-i}^{\mathrm{DD}}\big) = S_i^{\mathrm{DD}}$. Under $\widetilde{\alpha}_i - \widetilde{\beta}_i$, suppose chain $-i$ chooses the equilibrium inventory $\widetilde{q}_{-i}^{\mathrm{DD}}$, but chain i takes a suboptimal strategy q_i^s to maintain its original

service level: $F_i\big((1 - \widetilde{\alpha}_i + \widetilde{\beta}_i)q_i^s + \gamma_{-i}\widetilde{q}_{-i}^{\mathrm{DD}}\big) = S_i^{\mathrm{DD}}$. As we have shown, if $\gamma < 0$, q_{-i}^{DD} increases in $\alpha_i - \beta_i$. Hence, $\gamma_{-i}q_{-i}^{\mathrm{DD}} > \gamma_{-i}\widetilde{q}_{-i}^{\mathrm{DD}}$. Together with $1 - \alpha_i + \beta_i > 1 - \widetilde{\alpha}_i + \widetilde{\beta}_i > 0$, we have $q_i^s > q_i^{\mathrm{DD}}$. Under this suboptimal inventory level q_i^s, the profit of chain i is

$$\pi_i = 2(p_i - c_i)q_i^s - 2(p_i - v_i)\int_0^{F_i^{-1}(S_i^{\mathrm{DD}})} F_i(u)du$$

$$> 2(p_i - c_i)q_i^{\mathrm{DD}} - 2(p_i - v_i)\int_0^{F_i^{-1}(S_i^{\mathrm{DD}})} F_i(u)du \;=\; \pi_i^{\mathrm{DD}}.$$

Since a suboptimal inventory for chain i yields a higher profit, the optimal profit π_i^{DD} in the equilibrium must be even higher. The proves π_i^{DD} increases in $\alpha_i - \beta_i$ when $\gamma < 0$.

The monotonicity of π_i^{DD} in $\alpha_{-i} - \beta_{-i}$ can be shown using the same approach as above.

Proof of Theorem 4. The game dynamics is described and the equilibrium is proved in the paper. Here, we show the monotonicity results.

The effects of $\alpha_2 - \beta_2$ on the equilibrium follow the same lines of proof as in Theorem 2.

The effects of $\alpha_1 - \beta_1$ on the equilibrium follow the same lines of proof as in Theorem 3.

Proof of Corollary 5. Comparing S_i^{II} in (11) and S_i^{DD} in (18), and noting $\gamma_1\gamma_2 > 0$, we have the relation $S_i^{\mathrm{II}} < S_i^{\mathrm{DD}}$.

When $\gamma < 0$, comparing S_i^{DD} in (18) and S_i^* in (7), we obtain $S_i^{\mathrm{DD}} < S_i^*$.

When $\gamma > 0$, we have

$$S_i^* - S_i^{\mathrm{II}} = \frac{p_i - c_i - \frac{\gamma_i(p_{-i} - c_{-i})}{1 - \alpha_{-i} + \beta_{-i}}}{(p_i - v_i)(1 - \alpha_i + \beta_i - \frac{\gamma_1\gamma_2}{1 - \alpha_{-i} + \beta_{-i}})} - \frac{p_i - c_i}{(p_i - v_i)(1 - \alpha_i + \beta_i)}$$

$$= \frac{(p_i - c_i)(1 - \alpha_{-i} + \beta_{-i}) - \gamma_i(p_{-i} - c_{-i})}{(p_i - v_i)\eta} - \frac{(p_i - c_i)(1 - \alpha_{-i} + \beta_{-i}) - \gamma_1\gamma_2\frac{(p_i - c_i)}{(1 - \alpha_i + \beta_i)}}{(p_i - v_i)\eta}$$

$$= -\frac{\gamma_i\left[(p_{-i} - c_{-i}) - \frac{\gamma_{-i}(p_i - c_i)}{1 - \alpha_i + \beta_i}\right]}{(p_i - v_i)\eta}$$

$$= -\frac{\gamma_i S_{-i}^*}{1 - \alpha_i + \beta_i} \;<\; 0.$$

This proves $S_i^* < S_i^{\mathrm{II}}$ when $\gamma > 0$.

Proof of Corollary 7. The equilibrium inventory levels in (12), (19), and (27) from Theorems 2-4

are listed below:

$$q_i^{\mathrm{II}} = \frac{1}{\eta}\Big[(1 - \alpha_{-i} + \beta_{-i})F_i^{-1}(S_i^{\mathrm{II}}) - \gamma_{-i}F_{-i}^{-1}(S_{-i}^{\mathrm{II}})\Big],$$

$$q_i^{\mathrm{DD}} = \frac{1}{\eta}\Big[(1 - \alpha_{-i} + \beta_{-i})F_i^{-1}(S_i^{\mathrm{DD}}) - \gamma_{-i}F_{-i}^{-1}(S_{-i}^{\mathrm{DD}})\Big],$$

$$q_1^{\mathrm{DI}} = \frac{1}{\eta}\Big[(1 - \alpha_2 + \beta_2)F_1^{-1}(S_1^{\mathrm{II}}) - \gamma_2 F_2^{-1}(S_2^{\mathrm{DD}})\Big],$$

$$q_2^{\mathrm{DI}} = \frac{1}{\eta}\Big[(1 - \alpha_1 + \beta_1)F_2^{-1}(S_2^{\mathrm{DD}}) - \gamma_1 F_1^{-1}(S_1^{\mathrm{II}})\Big].$$

When $\gamma \neq 0$, we have $S_i^{\mathrm{DD}} > S_i^{\mathrm{II}}$, $i = 1, 2$, which immediately leads to the order of q_i^{II}, q_i^{DI}, and q_i^{DD} in the theorem.

Next, we prove the order of profits. Theorems 2-4 imply that

$$\pi_1^{\mathrm{II}} - \pi_1^{\mathrm{DI}} = 2(p_1 - c_1)(q_1^{\mathrm{II}} - q_1^{\mathrm{DI}}),$$

$$\pi_2^{\mathrm{DD}} - \pi_2^{\mathrm{DI}} = 2(p_2 - c_2)(q_2^{\mathrm{DD}} - q_2^{\mathrm{DI}}).$$

Thus, the order of π_1^{II} and π_1^{DI} is the same as that of q_1^{II} and q_1^{DI}, and the order of π_2^{DD} and π_2^{DI} is the same as that of q_2^{DD} and q_2^{DI}.

We next derive the order of π_2^{DI} and π_2^{II}. Define an auxiliary function,

$$\psi(x) \stackrel{\text{def}}{=} 2(p_2 - c_2)\frac{-\gamma_1 F_1^{-1}(S_1^{\mathrm{II}}) + (1 - \alpha_1 + \beta_1)x}{\eta} - 2(p_2 - v_2)\int_0^x F_2(u)du.$$

We have

$$\frac{d\psi(x)}{dx} = \frac{2(p_2 - c_2)(1 - \alpha_1 + \beta_1)}{\eta} - 2(p_2 - v_2)F_2(x), \tag{A.5}$$

$$\frac{d\psi(F_2^{-1}(S_2^{\mathrm{DD}}))}{dx} = \frac{2(p_2 - c_2)(1 - \alpha_1 + \beta_1)}{(1 - \alpha_1 + \beta_1)(1 - \alpha_2 + \beta_2) - \gamma_1\gamma_2} - \frac{2(p_2 - c_2)}{(1 - \alpha_2 + \beta_2) - \frac{\gamma_1\gamma_2}{1-\alpha_1+\beta_1}} = 0. \tag{A.6}$$

Equation (A.5) implies that $d\psi(x)/dx$ decreases in x, and it decreases to zero value when $x = F_2^{-1}(S_2^{\mathrm{DD}})$ in view of (A.6). Hence, $d\psi(x)/dx \geq 0$ for all $x \in [F_2^{-1}(S_2^{\mathrm{II}}), F_2^{-1}(S_2^{\mathrm{DD}})]$, and therefore, $\psi(x)$ increases in x, which leads to

$$\pi_2^{\mathrm{II}} = \psi(F_2^{-1}(S_2^{\mathrm{II}})) \leq \psi(F_2^{-1}(S_2^{\mathrm{DD}})) = \pi_2^{\mathrm{DI}}.$$

The order of π_1^{DI} and π_1^{DD} can be derived in the same logic with an auxiliary function:

$$\phi(x) \stackrel{\text{def}}{=} 2(p_1 - c_1)\frac{(1 - \alpha_2 + \beta_2)x - \gamma_2 F_2^{-1}(S_2^D)}{\eta} - 2(p_1 - v_1)\int_0^x F_1(u)du.$$

We can show that $\phi(x)$ increases in x for $\in [F_1^{-1}(S_1^{\mathrm{II}}), F_1^{-1}(S_1^{\mathrm{DD}})]$, and therefore,

$$\pi_1^{\mathrm{DI}} = \phi(F_1^{-1}(S_1^{\mathrm{II}})) \leq \phi(F_1^{-1}(S_1^{\mathrm{DD}})) = \pi_1^{\mathrm{DD}}.$$

Proof of Lemma 9.

(a) For any $x \le \widehat{x}$, if $G(x) = 0$, then $F(x) \ge G(x)$ is already satisfied. If $0 < G(x) \le G(\widehat{x}) = \widehat{y} < 1$, then by definition, we have $F^{-1}(\widehat{y}) - F^{-1}(G(x)) \ge G^{-1}(\widehat{y}) - G^{-1}(G(x))$, or equivalently $\widehat{x} - F^{-1}(G(x)) \ge \widehat{x} - x$, and thus $F^{-1}(G(x)) \le x$ or $G(x) \le F(x)$. Similarly, we can prove $F(x) \le G(x)$ for all $x \ge \widehat{x}$.

For $y \le \widehat{y}$, by definition, $F^{-1}(\widehat{y}) - F^{-1}(y) \ge G^{-1}(\widehat{y}) - G^{-1}(y)$, which implies $F^{-1}(y) \le G^{-1}(y)$. Similarly, for $y \ge \widehat{y}$, $F^{-1}(y) \ge G^{-1}(y)$.

(b) If there does not exist $\widehat{x}$ such that $F(\widehat{x}) = G(\widehat{x}) \in (0,1)$, then we have either $F(x) \ge G(x)$ or $G(x) \ge F(x)$ for all $x \in [0, \infty)$. Since $E[X] \ge \mathsf{E}[Y]$, only the later case stands. Thus $\int_a^\infty (G(x) - F(x))dx \ge 0$, for all $a \ge 0$. Hence $X \ge_v Y$.

If there exists $\widehat{x}$, such that $F(\widehat{x}) = G(\widehat{x}) \in (0,1)$, then part (a) applies. We have $G(x) - F(x) \le 0$ for $x \le \widehat{x}$ and $G(x) - F(x) \ge 0$ for $x \ge \widehat{x}$. Thus the function

$$s(a) \stackrel{\text{def}}{=} \int_{-a}^\infty G(x) - F(x)dx, \qquad a \in (-\infty, 0)$$

is increasing and then decreasing in a. $\mathsf{E}[X] \ge \mathsf{E}[Y]$ implies that $s(0) = \int_0^\infty G(x) - F(x)dx \ge 0$. Together with $s(-\infty) = 0$, we have $s(a) \ge 0$ for all $a \in (-\infty, 0)$. This implies $X \ge_v Y$.

(c) Integrating by parts, we have $\int_0^{F^{-1}(x)} F(u)du = xF^{-1}(x) - \int_0^x F^{-1}(u)du$. Thus,

$$\int_0^{F^{-1}(x)} F(u)du - \int_0^{G^{-1}(x)} G(u)du = \int_0^x \left[(F^{-1}(x) - F^{-1}(u)) - (G^{-1}(x) - G^{-1}(u)) \right] du \ge 0,$$

where the last inequality is due to the fact that $X \ge_d Y$ so that the integrand is always nonnegative. Similarly, we can prove $\int_{F^{-1}(y)}^\infty (1 - F(u))du \ge \int_{G^{-1}(y)}^\infty (1 - G(u))du$.

Proof of Theorem 10. We only need to prove $g_i(y, F_i) \le g_i(y, G_i)$, for all $y \in (0,1)$. Since $\mathsf{E}[\xi_{ij}^F] = \mathsf{E}[\xi_{ij}^G]$, there must exist $\widehat{x}$ such that $F(\widehat{x}) = G(\widehat{x}) = \widehat{y}$.

From Lemma 9 (a) and (c), when $y \le \widehat{y}$, $F_i^{-1}(y) \le G_i^{-1}(y)$ and $\int_0^{F_i^{-1}(y)} F_i(u)du \ge \int_0^{G_i^{-1}(y)} G_i(u)du$. These together imply that

$$g_i(y, F_i) \le g_i(y, G_i) \qquad \text{for } y \le \widehat{y}.$$

We note that $\mathsf{E}[\xi_{ij}^F] = \mathsf{E}[\xi_{ij}^G]$ implies that

$$\int_0^\infty (1 - F_i(u))du = \int_0^\infty (1 - G_i(u))du,$$

or equivalently,

$$g_i(y, F_i) + (1 - \gamma_i)F_i^{-1}(y) + \int_{F_i^{-1}(y)}^{\infty} (1 - F_i(u))du$$

$$= g_i(y, G_i) + (1 - \gamma_i)G_i^{-1}(y) + \int_{G_i^{-1}(y)}^{\infty} (1 - G_i(u))du$$

When $y \geq \widehat{y}$, from Lemma 9 (a) and (c), we have $F^{-1}(y) \geq G^{-1}(y)$ and $\int_{F^{-1}(y)}^{\infty} (1 - F_i(u))du \geq \int_{G_i^{-1}(y)}^{\infty} (1 - G_i(u))du$, Thus $g_i(y, F_i) \leq g_i(y, G_i)$ for $y \geq \widehat{y}$.

Together, we have $g_i(y, F_i) \leq g_i(y, G_i)$ for all $y \in (0, 1)$. Hence,

$$\pi^{\mathrm{X}}(F_1, F_2) \leq \pi^{\mathrm{X}}(G_1, G_2).$$

To prove the second part regarding to the individual supply chain profit, we express supply chain's profit as

$$\pi_i^{\mathrm{X}}(F_1, F_2) = 2(p_i - v_i)\widetilde{g}_i(S_i^{\mathrm{X}}, F_i) - \frac{2(p_i - c_i)\gamma_{-i}}{\eta} F_{-i}^{-1}(S_{-i}^{\mathrm{X}}),$$

where $\widetilde{g}_i(S_i^{\mathrm{X}}, F_i) = S_i^{\mathrm{DD}} F_i^{-1}(S_i^{\mathrm{X}}) - \int_0^{F_i^{-1}(S_i^{\mathrm{X}})} F_i(u)du$ is similar to $g_i(y, F_i)$ and is also decreasing when F becomes more dispersed while keeping mean constant. Hence the result.

While if F_{-i} becomes more dispersed, the direction of the change of π_i^{X} is unclear. It depends on how the more dispersed distribution is shaped. In particular, it depends on whether $\widehat{x} \leq S_{-i}^{\mathrm{X}}$.

Proof of Theorem 11.

$$\left| \pi^{\mathrm{X}_1}(F_1, F_2) - \pi^{\mathrm{X}_2}(F_1, F_2) \right| = \left| \sum_i 2(p_i - v_i) \int_{S_i^{\mathrm{X}_2}}^{S_i^{\mathrm{X}_1}} \frac{dg_i(y; F_i)}{dy} dy \right|$$

$$= \sum_i 2(p_i - v_i) \int_{S_i^{\mathrm{X}_2}}^{S_i^{\mathrm{X}_1}} \left| \frac{dg_i(y; F_i)}{dy} \right| dy$$

$$= \sum_i 2(p_i - v_i) \int_{S_i^{\mathrm{X}_2}}^{S_i^{\mathrm{X}_1}} |\gamma_i - y| \frac{dF_i^{-1}(y)}{dy} dy$$

$$\geq \sum_i 2(p_i - v_i) \int_{S_i^{\mathrm{X}_2}}^{S_i^{\mathrm{X}_1}} |\gamma_i - y| \frac{dG_i^{-1}(y)}{dy} dy$$

$$= \left| \pi^{\mathrm{X}_1}(G_1, G_2) - \pi^{\mathrm{X}_2}(G_1, G_2) \right|,$$

where the second equality is due to the fact that $\int_{S_i^{\mathrm{X}_2}}^{S_i^{\mathrm{X}_1}} \frac{dg_i(y; F_i)}{dy} dy$ are of the same signs for $i = 1, 2$, and the inequality follows from (33).